QUIZ 17 SOLUTIONS: LESSON 24
OCTOBER 30, 2017

Write legibly, clearly indicate the question you are answering, and put a box or circle around your final answer. If you do not clearly indicate the question numbers, I will take off points. Write as much work as you need to demonstrate to me that you understand the concepts involved. If you have any questions, raise your hand and I will come over to you.

1. [10 pts] What is the minimum surface area of a cylinder with a volume of 16π?

   Solution: The formula for the surface area of a cylinder is
$$SA = 2\pi r^2 + 2\pi r h.$$
Its volume is given by
$$16\pi = V = \pi r^2 h.$$
Solving for h, we see that
$$\frac{16}{r^2} = h.$$
Substituting this into our formula for surface area, we want to minimize the function
$$SA = 2\pi r^2 + 2\pi r\left(\frac{16}{r^2}\right) = 2\pi r^2 + \frac{32\pi}{r}.$$
Note that this is a function of a single variable, so we apply methods from Calc 1.

   Taking the derivative with respect to r, we get
$$\frac{dSA}{dr} = 4\pi r - \frac{32\pi}{r^2}.$$
We set this equal to 0 and solve. So we write
$$0 = 4\pi r - \frac{32\pi}{r^2}$$
$$\Rightarrow \quad \frac{32\pi}{r^2} = 4\pi r$$
$$\Rightarrow \quad 32\pi = 4\pi r^3$$
$$\Rightarrow \quad 8 = r^3$$
Hence, our critical point is
$$r = \sqrt[3]{8} = 2.$$
Note 0.1. Technically, $r = 0$ is also a critical point because the derivative does not exist there. However, we can't have a volume of 8π if $r = 0$ so we discount it.

Since

$$\frac{dSA}{dr}(1) = 4\pi(1) - \frac{32\pi}{(1)^2} = 4\pi - 32\pi < 0$$

and

$$\frac{dSA}{dr}(4) = 4\pi(4) - \frac{32\pi}{(4)^2} = 16\pi - 2\pi > 0,$$

we conclude this is indeed a minimum (you didn't need to check this step).

Therefore, the minimal surface area is

$$SA(2) = 2\pi(2)^2 + \frac{32\pi}{(2)} = 8\pi + 16\pi = \boxed{24\pi}.$$